

Postdoctoral project
**Numerical study of hybrid resonances in cold
plasma**

Context

Time-harmonic wave propagation in cold magnetized plasmas is described by the Maxwell equations with a frequency-dependent spatially-varying tensor of dielectric permittivity. Interaction of the exterior magnetic field with media may lead to plasma heating. Mathematically, plasma heating is related to the presence of the singularities in the electric field (which is no longer L^2 , unlike in the case of electromagnetic wave propagation in dielectrics). Rigorous mathematical theory of hybrid resonances in plasmas was initiated by Bruno Després and his co-workers, see [1, 2, 3, 4]. They have shown that the corresponding singularities can be studied by examining the limiting absorption solution to a certain boundary-value problem. In the simplest setting, this problem is described by a second-order PDE with a sign-changing coefficient in the principal part of the operator. Unlike in the case of plasmonic applications, this coefficient is typically smooth, and the corresponding problem is well-posed in a non-standard functional framework. This naturally leads to questions of the choice of the appropriate variational formulation, as well as of the design of the numerical methods to solve such problems.

While some of these questions had been answered in recent years, the mathematical and numerical analysis of the realistic two-dimensional Maxwell equations, as well as full three-dimensional Maxwell system is still in its infancy stage (see e.g. [2] for a related study).

Postdoctoral project

This **2-year** postdoctoral research subject is a continuation of the PhD thesis of Étienne Peillon [5], see also [6]. It is dedicated to the design and analysis of alternative variational formulations, and their extension to the case of the Maxwell equations.

We aim at tackling the following questions:

- design of an alternative mixed formulation for the solution of the scalar 2D problem $\operatorname{div}(\alpha \nabla u) = 0$ with α changing its sign across a regular interface, in a well-adapted functional framework. The key idea is a re-interpretation of this problem as a special transmission problem, with well-chosen trace operators, see [7].
- implementation of this formulation, its mathematical and numerical analysis.

- extension to the realistic case of 2D Maxwell equations, where the tensor of dielectric permittivity is no longer diagonal.

Keywords: plasma resonance, singular solutions, Maxwell equations

Requirements: The successful candidate has a PhD in numerical analysis, scientific computing, analysis of PDEs or related fields. They have strong programming skills, knowledge of numerical analysis and computational mathematics, and, possibly, analysis of PDEs .

Supervision: The postdoctoral research subject will be supervised by Patrick Ciarlet and Maryna Kachanovska. The successful candidate will be based at ENSTA in Palaiseau.

Starting date: Any date starting from the 1st of February of 2025, until the position is filled.

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References

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