Post-doctoral research project: efficient resolution of time-domain transmission problems in the presence of unbounded interfaces

Context

Acoustic and electromagnetic wave propagation in infinite composite media with piecewise constant characteristics occurs in many applications, such as scattering from ocean surfaces (e.g. fig. 1) and light/sound propagation in waveguides (e.g. fig. 2). While boundary integral equation methods are known for being very efficient in the solution of such scattering problems by *bounded* obstacles (they provide both a dimensional reduction, as well as a natural way to handle the unbounded computational domain), their practical application in the presence of infinite interfaces is significantly more complicated. The main reason is that when the interfaces themselves are infinite — and not only the domain — the boundary integral equation is posed on unbounded curves/surfaces which must be truncated. The non-local nature of boundary integral operators makes this truncation subtle.

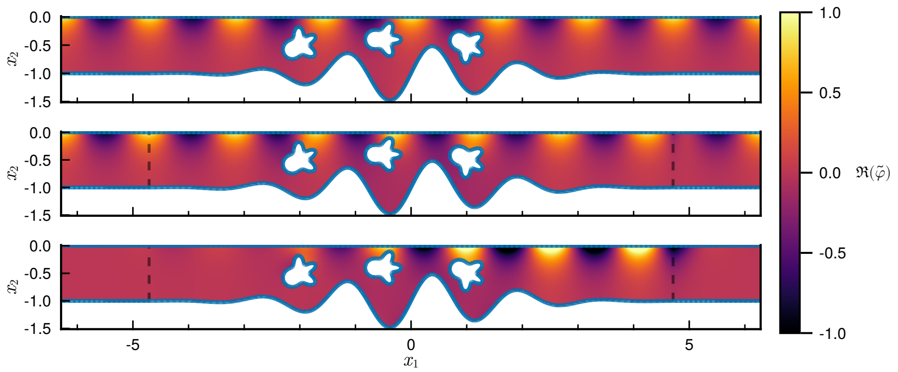

Figure 1: Time-harmonic water problem. Incident field (top), total field (middle), and scattered field (bottom) for three jellyfish-like over a variable topography.

To address time-harmonic problems in this context, as far as our knowledge goes, there are two main methods available: the use of special quadrature rules (as seen in methods like the Windowed Green function method [1]), and deforming the integral contour within the complex plane (which may involve applying the so-called perfectly matched layer method [2]). The objective of this

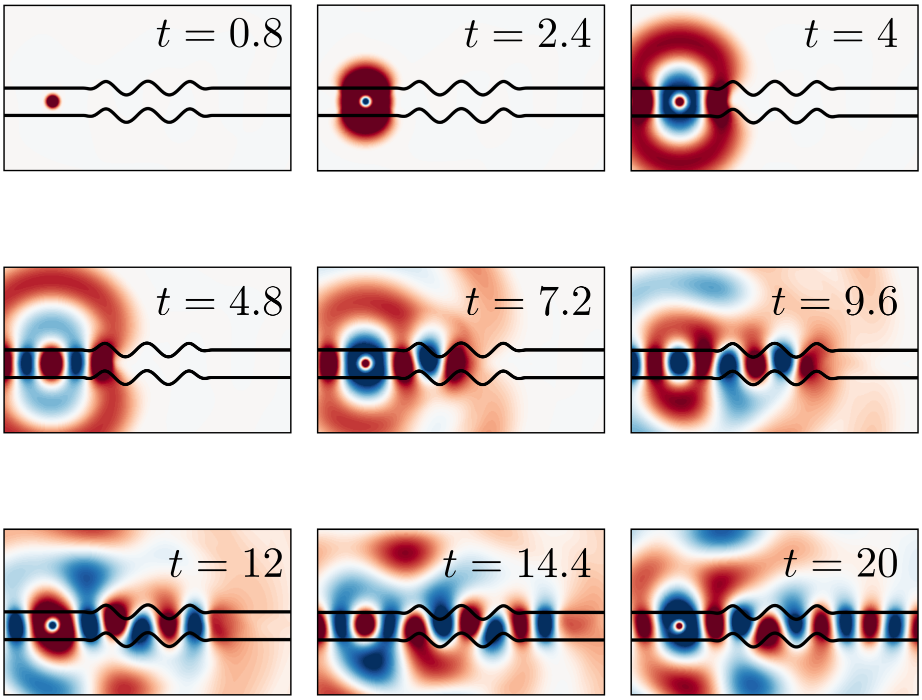

Figure 2: Time-evolution of an acoustic pulse inside a leaky waveguide.

postdoctoral research subject is to explore the latter technique in the time domain, which, to the best of our knowledge, is an unexplored and novel research direction.

Project

We propose to begin our investigation with the simplest problem: scattering by an infinite interface in two dimensions (see fig. 3 for a schematic description). Our objective is to combine the perfectly matched layer method, typically used for volume problems, with time-domain boundary integral equations, inspired by the work [3] for time-harmonic problems. This combination leads to new integral equation formulations with modified integral kernels. These modified kernels exhibit exponential decay in the direction tangent to the interface (and away from the data's support). This property enables the efficient truncation of the resulting surface integrals in numerical computations with minimal loss of precision; more precisely, the truncation errors decrease *exponentially* with the increase of the size of the truncation layer.

The primary challenge to be addressed lies in discretizing the resulting formulation in time. While in 2D, the corresponding kernels are analytically known in the time domain, this is no longer the case in three dimensions. Hence, it is natural to employ the convolution quadrature method [4], which only requires knowledge of the integral kernels in the Laplace domain.

The first goals of the current project are to (i) understand the theoretical foundations of the proposed method, (ii) implement the technique in the form of

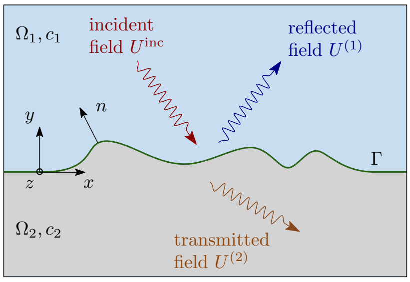

Figure 3: Schematic of a typical scattering problem in the presence of infinite interfaces. Given an incident field, one seeks to find the reflected and transmitted fields that solve both the partial differential equation, as well as certain transmission conditions at the interface.

a computer program in either Matlab or Julia, and (iii) evaluate the numerical performance of this methodology compared to the existing work described in [5].

Perspectives

In future, we would like to address the following points:

- the well-posedness of the corresponding formulations;
- possibility to exploit the sparseness of the matrices for more efficient implementation of the CQ method for this problem, cf. e.g. [6];
- a rigorous truncation error analysis, both in frequency as well as in time-domain (stemming from the truncation of the integrals).

Requirements

The successful candidate has a PhD in numerical analysis, scientific computing, analysis of PDEs, boundary integral equations or related fields.

Since this project is funded by the French Government Defense Agency, the EU/Norvegian/Swiss citizenship is highly desirable.

Keywords: numerical implementation, boundary integral equations, convolution quadrature

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